UNITED STATES PATENT APPLICATION

OF

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FOR

COMPUTER GRAPHICS SYSTEM AND COMPUTER-IMPLEMENTED METHOD FOR GENERATING

COARSE-LEVEL MESHES FOR MULTI-RESOLUTION SURFACE EDITING

FIELD OF THE INVENTION

The invention relates generally to the field of computer graphics, and more specifically to generation of meshes representing surfaces of objects. The invention specifically provides a computer graphics system and computer-implemented method for generating a coarse-level mesh from a fine-level mesh, which may be used in multi-resolution surface editing.

BACKGROUND OF THE INVENTION

Generally, in computer graphics, objects are represented as surfaces, with the surfaces being represented by meshes. A mesh consists of a set of vertices, or points in three-dimensional space, interconnected by edges. The edges define polygonal faces, which may be in the form of triangles, quadrilaterals, and so forth. In some computer graphic operations, it is desirable to generate a representation of a surface at a finer resolution than a current representation. There are several popular methodologies for generating a representation of a surface at a finer resolution than a current representation, including a Catmull-Clark surface subdivision methodology and a Loop surface subdivision methodology. In other operations, it can also be desirable to generate from a mesh representing a surface at one level of resolution, a mesh at a coarser resolution. A methodology for generating a representation of a surface at a coarser resolution than a current representation, and a methodology for generating such a surface representation, based on the Loop methodology, is described in D. Zorin, et al., "Interactive Multiresolution Mesh Editing, SIGGRAPH 1997, Computer Graphics Proceedings, pp. 259-268, particularly p. 262.

The methodology described in Zorin relies on a method for smoothing triangular meshes that is described in G. Taubin, "A signal processing approach to fair surface design," SIGGRAPH 1995, Computer Graphics Conference Proceedings, pp. 351-358. Taubin's smoothing methodology consists of two linear filtering steps, the first being a low-pass filter step, followed by a high-pass filter step. The high-pass filter step is used to cancel out a mesh-shrinking effect that is induced by the low-pass filter step. The filtering operations described by Taubin have the problem that, as a consequence of the linearity of the filtering procedure, artifacts referred to as Gibbs ripples are

1 introduced in the smoothed mesh as a result of discontinuities in the input mesh. When used in

2 generating a coarse level mesh, Taubin's filtering produces Gibbs ripples at the places on the surface

3 where the fine level mesh has sharp variations. Although Taubin's smoothing methodology may be

4 configured with the help of parameters, no filter can play the role of an inverse of the Loop

5 subdivision rule. Accordingly, in a multi-resolution representation of a Loop subdivision surface,

6 there will be redundant detail information.

7 **SUMMARY OF THE INVENTION**

8 The invention provides a new and improved system and method for generating a coarse-level

9 mesh from a fine-level mesh, which may be used in multi-resolution surface editing.

10 In brief summary, the invention provides an arrangement for generating a coarse ("j") level

11 mesh representing a surface, from a finer ("j+1") level mesh surface representation. The

12 arrangement includes an indicator value generator and a coarse level mesh generator. The indicator

13 value generator, for respective ones of the points in the finer level mesh surface representation,

14 evaluates an indicator function, the value indicating whether a subdivision-inverse filter

15 methodology or a least-squares optimization methodology is to be used to determine a position for

16 the corresponding point in the coarse level mesh representation. The coarse level mesh generator

17 determines, for each of the points that is to be provided in the coarse level mesh representation, a

18 position in response to the position of the corresponding point in the finer level mesh representation,

19 in accordance with the one of the subdivision-inverse filter methodology or least-squares

20 optimization methodology indicated by the indicator value generated by the indicator value

21 generator.

22 **BRIEF DESCRIPTION OF THE DRAWINGS**

23 This invention is pointed out with particularity in the appended claims. The above and

24 further advantages of this invention may be better understood by referring to the following

25 description taken in conjunction with the accompanying drawings, in which:

1   FIG. 1 depicts a computer graphics system including an arrangement for generating a coarse

2   level mesh representation of a surface from a finer level mesh of a surface, constructed in accordance

3   with the invention;

4   FIG. 2 depicts a mesh representing a surface to which the Loop surface subdivision

5   methodology has been applied;

6   FIG. 3 depicts a mesh representing a surface to which the Catmull-Clark surface subdivision

7   methodology has been applied;

8   FIGS. 4 and 5 depict diagrams useful in understanding operations performed by the fine-to-

9   coarse level mesh generating arrangement;

10   FIG. 6 depicts a flow chart describing operations performed by the fine-to-coarse level mesh

11   generating arrangement in connection with triangular meshes; and

12   FIG. 7 depicts a flow chart describing operations performed by the fine-to-coarse level mesh

13   generating arrangement in connection with quadrilateral meshes.

DETAILED DESCRIPTION OF AN ILLUSTRATIVE EMBODIMENT

15   FIG. 1 depicts a computer graphics system 10 including an arrangement for generating a

16   coarse level mesh representation of a surface from a finer level mesh of a surface, constructed in

17   accordance with the invention. With reference to FIG. 1, the computer graphics system includes a

18   processor module 11, one or more operator input devices 12 and one or more display devices 13.

19   The display device(s) 13 will typically comprise a frame buffer, video display terminal or the like,

20   which will display information in textual and/or graphical form on a display screen to the operator.

21   The operator input devices 12 for a computer graphics system 10 will typically include a pen 14

22   which is typically used in conjunction with a digitizing tablet 15, and a trackball or mouse device

23   16. Generally, the pen 14 and digitizing tablet will be used by the operator in several modes. In one

24   mode, the pen 14 and digitizing tablet are used to provide updated shading information to the

25   computer graphics system. In other modes, the pen and digitizing tablet are used by the operator to

26   input conventional computer graphics information, such as line drawing for, for example, surface

1   trimming and other information, to the computer graphics system 10, thereby to enable the system

2   10 to perform conventional computer graphics operations. The trackball or mouse device 16 can be

3   used to move a cursor or pointer over the screen to particular points in the image at which the

4   operator can provide input with the pen and digitizing tablet. The computer graphics system 10 may

5   also include a keyboard (not shown) which the operator can use to provide textual input to the

6   system 10.

7   The processor module 11 generally includes a processor, which may be in the form of one

8   or more microprocessors, a main memory, and will generally include one a mass storage subsystem

9   including one or more disk storage devices. The memory and disk storage devices will generally

10   store data and programs (collectively, "information") to be processed by the processor, and will store

11   processed data which has been generated by the processor. The processor module includes

12   connections to the operator input device(s) 12 and the display device(s) 13, and will receive

13   information input by the operator through the operator input device(s) 12, process the input

14   information, store the processed information in the memory and/or mass storage subsystem. In

15   addition, the processor module can provide video display information, which can form part of the

16   information obtained from the memory and disk storage device as well as processed data generated

17   thereby, to the display device(s) for display to the operator. The processor module 11 may also

18   include connections (not shown) to hardcopy output devices such as printers for facilitating the

19   generation of hardcopy output, modems and/or network interfaces (also not shown) for connecting

20   the system 10 to the public telephony system and/or in a computer network for facilitating the

21   transfer of information, and the like.

22   The invention provides an arrangement for generating a coarse-level mesh from a finer level

23   mesh. In the following, the fine level mesh will continue to be identified by index "j+1," and the

24   coarse level mesh will continued to be identified by index "j." Before describing the operations

25   performed by the fine-to-coarse mesh generating arrangement in connection with generating a

26   coarse level mesh from a current representation, it would be helpful to describe two surface

27   subdivision methodologies, namely the aforementioned Loop surface subdivision methodology and

28   a Catmull-Clark surface subdivision methodology, which are used to generate a finer ("j+1) level

29   mesh from a coarse ("j") level mesh. The invention provides an arrangement for essentially

1   performing, in the case of a triangular mesh, operations that generally provide the inverse of Loop's

2   surface subdivision methodology, and, in the case of a quadrilateral mesh, operations that generally

3   provide the inverse of the Catmull-Clark surface subdivision methodology.

4   Loop's surface subdivision methodology will be described in connection with FIG. 2.

5   Generally, in Loop's surface subdivision methodology, each triangular face in the original mesh is

6   split into a plurality of subface, the subfaces defining the finer level mesh. The vertices of the finer

7   level mesh are positioned using weighted averages of the positions of the vertices in the original

8   mesh. More specifically, and with reference to FIG. 2, a mesh 20 at a fine "j" level is depicted that

9   includes a vertex v_q 21(0) located at position $c^j(q)$, and a plurality "K" of surrounding points $v_q(1)$

10   through $v_q(K)$ (generally, $v_q(k)$) 21(1) through 21(K) (generally identified by reference numeral

11   21(k)) located at positions at respective positions $c^j(1)$ through $c^j(K)$, the points $v_q(k)$ 21(k) being

12   those points in the mesh 20 that are connected to vertex v_q by edges. ("K," the number of points

13   connected to vertex v_q will sometimes be referred to as the vertex's "valence.") Denoting the set of

14   the indices of the points $v_q(k)$ in the mesh that are connected to vertex v_q by N(q,j), a mesh at the

15   next higher fineness level "j+1" corresponds to vertex v'_q 22(0) located at position $c^{j+1}(q)$ and a set

16   of surrounding points $v'_q(l)$ 22(l) at located at positions $c^{j+1}(l)$ and connected to the vertex v'_q 22(0).

17   The finer level mesh is constructed by providing the vertex v'_q 22(0) at a position $c^{j+1}(q)$ that is

18   determined by

19

$$c^{j+1}(q) \;=\; \left(1 - a(K)\right) c^j(q) + \frac{a(K)}{K} \sum_{k \in N(q,J)} c^j(k) \tag{1}$$

20   and points $v'_q(l)$ 22(l) at positions $c^{j+1}(l)$ that are determined by

21

$$c^{j+1}(l) \;=\; \frac{3}{8}\left[c^j(q) + c^j(k)\right] + \frac{1}{8}\left[c^j(k-1) + c^j(k+1)\right], k = 1,\ldots,K \tag{2},$$

22   where the weighting factor a(K) is given by

$$a(K) = \frac{5}{8} - \left(\frac{3 + 2\cos\left(\frac{2\pi}{K}\right)}{8} \right)^2 \tag{3}.$$

1

2 Equations (1) and (2) are applied by selecting each point in the mesh at level "j" as a vertex, to

3 provide points for the mesh at level "j+1." Loop's surface subdivision methodology can be applied

4 recursively to provide a mesh at any desired degree of fineness. It will be appreciated that equation

5 (2) will provide that the point $v'_q(1)$ 22(1) that is created when the methodology is applied to point

6 v_q 21(0) as the vertex will be in the same position when the methodology is applied to point $v_q(k)$

7 21(k) as the vertex.

It will be appreciated that, the Loop surface subdivision methodology provides

8

9 (i) one new point, at location $c^{j+1}(1)$ in the finer ("j+1") level mesh, for, and generally

10 somewhere between, each pair of points in the coarse ("j") level mesh, and

11 (ii) a relocated point, at location $c^{j+1}(q)$ in the finer ("j+1") level mesh for, and generally

12 somewhat near, each point in the coarse ("j") level mesh,

13 which are interconnected to form the finer ("j+1) level mesh. Accordingly, in performing the

14 inverse, that is, processing a finer ("j+1") level mesh to generate a coarse ("j") level mesh, the

15 arrangement processes the points in the finer ("j+1") level mesh to remove, for each pair of

16 connected points in the finer ("j+1") level mesh, and determines a position for the remaining point,

17 which will be present in the coarse ("j") level mesh, based on the positions of surrounding points in

18 the finer ("j+1") level mesh. The operations performed by the arrangement in determining the

19 position of the point that is to remain in the coarse ("j") level mesh will be described below.

20 The Catmull-Clark subdivision methodology produces smooth surfaces using a small number

21 of neighboring vertices. The Catmull-Clark surface subdivision methodology will be described in

1   connection with FIGS. 3A through 3E. With reference to FIG. 3A, that FIG. depicts a mesh 30

2   consisting of four quadrilaterals 31(1) through 31(4). Each quadrilateral is referred to as a face. The

3   quadrilaterals are defined by points 32(0), which is common to all of the quadrilaterals 31(1) through

4   31(4), and other points 32(1) through 32(8). Generally, the Catmull-Clark surface subdivision

5   methodology is performed in a series of iterations, including

6   (i) a face point generation iteration, in which, for each face, a face point 33(1) through 33(4)

7   is generated as the average of the points 32(p) defining the respective face (reference FIG. 3B);

8   (ii) an edge point generation iteration in which, for each edge, a new edge point 34(1)

9   through 34(4) is generated as the average of the midpoint of the original edge with the two new face

10   points of the faces adjacent to the edge (reference FIG. 3C; the face points generated during the face

11   point generation iteration are shown as circles);

12   (iii) a vertex point iteration in which a vertex point 35 is generated in relation to the positions

13   of the new face points generated in iteration (i) for faces that are adjacent to the original vertex point,

14   the positions of the midpoints of the original edges that are incident on the original vertex point, the

15   position of the original vertex point, and the valence of the original vertex point (reference FIG. 3D;

16   the face and edge points generated during the face point and edge point generation iterations are

17   shown as circles); and

18   (iv) a mesh connection step (reference FIG. 3E) in which:

19   (a)   each new face point is connected to the edge points of the edges defining the original

20   face; and

21   (b)   each new vertex point is connected to the new edge points of all original edges

22   incident on the original vertex point.

23   More specifically, in the Catmull-Clark subdivision methodology, denoting, for a vertex v_q

24   at location $c^j(q)$,

25   (i) the set of indices of neighboring points $c^j(l)$ that are connected thereto (for example, for

26   point 32(0), point 32(2) and 32(8)) by $N_e(q,j)$, and

1   (ii) the set of indices of second-order neighboring points that are opposite to vertex $c^j(q)$ with

2   respect to the level "j" faces (for example, for point 32(0), point 32(1)) that are incident with vertex

3    $c^j(q)$ by $N_f(q,j)$

4   a mesh at the next higher fineness level "j+1" is constructed as follows. In the face point generation

5   iteration, the face points are generated and located at positions $c^{j+1}(m_i)$ determined as follows

6

$$c^{j+1}(m_i) = \frac{1}{4}\left(c^j(q) + c^j(l_i) + c^j(l_{i+1}) + c^j(r_i)\right),$$

$$m_i \in N_f(q, j+1), l_i, l_{i+1} \in N_e(q, j), r_i \in N_f(q, j) \tag{4}$$

In the edge point generation iteration, the edge points are generated and located at positions $c^{j+1}(l_i)$
determined as follows:

$$c^{j+1}(l_i) = \frac{1}{4}\left(c^j(q) + c^j(k_i) + c^{j+1}(m_{i-1}) + c^{j+1}(m_i)\right),$$

$$l_i \in N_e(q, j+1), k_i \in N_e(q, j), m_{i-1}, m_i \in N_f(q, j+1) \tag{5}$$

In the vertex point generation iteration, the new vertex points are generated and located at positions
as follows:

12

$$c^{j+1}(q) = \frac{K-2}{K}c^j(q) + \frac{1}{K^2}\sum_{i=0}^{K-1}c^j(l_i) + \frac{1}{K^2}\sum_{i=0}^{K-1}c^{j+1}(m_i)$$

$$l_i \in N_e(j+1, q), m_i \in N_f(j+1, q) \tag{6}$$

13   In terms of only points c^j, that is substituting for the last term in equation (6),

$$c^{j+1}(q) = \left(1 - \frac{7}{4K}\right)c^j(q) + \frac{3}{2}\frac{1}{K^2}\sum_{l\in N_e(q,j)}c^j(l) + \frac{1}{4K^2}\sum_{m\in N_f(q,j)}c^j(m) \tag{7}$$

1

2   Generally, for quadrilateral meshes, the arrangement makes use of the Catmull-Clark

3 methodology, except for the case of K=3 in equation (6). In that case, the arrangement makes use

4 of

5
$$c^{J+1}(q) \;=\; \left(1 - 8\gamma\right) c^{J}(q) + 4\gamma \, \frac{1}{K} \sum_{l \in N_{e}(q,J)} c^{J}(l) + 4\gamma \, \frac{1}{K} \sum_{m \in N_{f}(q,J+1)} c^{J+1}(m) \qquad \textbf{(8)},$$

6 where $\gamma = \dfrac{3}{38}$. Equation (8) would correspond to the Catmull-Clark methodology (equation (6))

7 with $\gamma = \dfrac{1}{4K}$, except that, for K=3, $\gamma = \dfrac{3}{38}$ instead of $\dfrac{1}{12}$. In terms of only points c^{j} (compare

8 equation (7))

9
$$c^{J+1}(q) \;=\; \left(1 - 7\gamma\right) c^{J}(q) + 6\gamma \, \frac{1}{K} \sum_{l \in N_{e}(q,J)} c^{J}(l) + \gamma \, \frac{1}{K} \sum_{m \in N_{f}(q,J)} c^{J}(m) \qquad \textbf{(9)}.$$

10   As is apparent from the above discussion, the Catmull-Clark surface subdivision

11 methodology applied to a quadrilateral mesh at the coarse ("j") level will produce a quadrilateral

12 mesh at the finer ("j+1") level having the same number of points as the mesh at the coarse ("j") level,

13 but the points in the finer ("j+1") level may be at different positions than in the coarse ("j") level.

14 Accordingly, in performing the inverse, that is, processing a finer ("j+1") level mesh to generate a

15 coarse ("j") level mesh, the arrangement processes the points in the finer ("j+1") level mesh

16 determine a position for each point in the finer ("j+1") level mesh based on the positions of

17 surrounding points in the finer ("j+1") level mesh. The operations performed by the arrangement

18 in determining the position of the point in the coarse ("j") level mesh will be described below.

19   As noted above, the invention provides an arrangement for generating a coarse ("j) level

20 mesh from a finer ("j+1") level mesh. In generating a coarse mesh from a fine mesh, an indicator

1 function is used locally in connection with each point that is being processed as a vertex to determine

2 whether, around the respective point, the fine mesh can be derived by subdivision of a coarser mesh

3 using a subdivision inverse filter. If so, the value of the indicator function, which will be described

4 below, will be "zero" or close to "zero." On the other hand, if the fine mesh cannot be derived by

5 subdivision of a coarser mesh in the region around the respective vertex using a subdivision inverse

6 filter, the value of the indicator function will not be close to "zero," and in that case the coarse mesh

7 will be generated using an approximate solution of a least-squares problem.

8   The indicator function is constructed for each point, selected as a vertex v'_q, using the value

9 of the Laplacian $L(k,j+1)$ of the positions of the point and of the other points in its neighborhood in

10 the finer (j+1) level mesh. For triangular meshes, in the one-dimensional case, for example, for a

11 vertex v'_q on a boundary, crease line or the like in the finer ("j+1") level mesh, the Laplacian for the

12 vertex is

$$L(k,j+1) \;=\; \frac{1}{2}\left[c^{j+1}(k-1) + c^{j+1}(k+1)\right] - c^{j+1}(k) \tag{10},$$

14 where $c^{j+1}(k)$ is the position of the vertex v'_q in the finer ("j+1) level mesh for which the Laplacian

15 is being generated, and $c^{j+1}(k-1)$ and $c^{j+1}(k+1)$ are the positions of the neighboring points $v'_q(k-1)$ and

16 $v'_q(k+1)$ in the fine (j+1) level mesh. In the two-dimensional case, that is, in the case of a vertex v'_q

17 other than on a boundary, crease line or the like, for a regular vertex, that is, for a vertex for which

18 the valence "K" is equal to "six," the Laplacian is

19

$$L(k,j+1) \;=\; \frac{1}{6}\left(\sum_{l \in N(k,j+1)} c^{j+1}(l)\right) - c^{j+1}(k) \tag{11},$$

20 where the factor "1/6" reflects the valence of the vertex. Also in the two-dimensional case, for an

21 irregular vertex, that is, for a vertex for which the valence "K" is not equal to "six," which may be

22 the case at, for example, edges of the mesh, the Laplacian is

$$L(k, j+1) \;=\; \rho\left[\frac{1}{K}\sum_{l \in N(k, J+1)} c^{J+1}(l) - c^{J+1}(k)\right] \tag{12},$$

1

2   where

3

$$\rho \;=\; -\frac{3 + 8\,a(K)}{3\big(-5 + 8\,a(K)\big)} \tag{13},$$

4   where a(K) is defined in equation (3) above.

5   For quadrilateral meshes, in the one-dimensional case, that is, for a vertex v'_q on a boundary,

6   crease line, or the like in the finer ("j+1") level mesh, the Laplacian L(k,j+1) for the vertex v'_q is

$$L(k, j+1) \;=\; \frac{1}{2}\big(c^{J+1}(k-1) + c^{J+1}(k+1)\big) - c^{J+1}(k) \tag{14}$$

8   where $c^{j+1}(k)$ is the position of the vertex v'_q for which the Laplacian is being generated in the finer

9   ("j+1") level mesh, and $c^{j+1}(k-1)$ and $c^{j+1}(k+1)$ are the positions of the neighboring points in the finer

10 (j+1) level mesh. In the two-dimensional case, that is, in the case of a vertex v'_q other than on a

11 boundary, crease line or the like, two Laplacians are used, namely a Laplacian $L_e(k,j+1)$ using the

12 position $c^{j+1}(k)$ of the vertex v'_q and the positions of the points whose indices are in the set $N_e(q,j)$,

13 and a Laplacian $L_f(k,j+1)$ using the position $c^{j+1}(k)$ and the positions of the vertices whose indices

14 are in the set $N_f(q,j)$, as follows

15

$$L_e(k, j+1) \;=\; \frac{1}{K}\left(\sum_{l \in N_e(k, J+1)} c^{J+1}(l)\right) - c^{J+1}(k) \tag{15}$$

16 and

$$L_f\left(k, j+1\right) = \frac{1}{K}\left(\sum_{l \in N_f\left(k, J+1\right)} c^{j+1}(l)\right) - c^{j+1}(k) \tag{16}.$$

1

As will be described below, the arrangement makes use of values of both Laplacians $L_e(k,j+1)$ and $L_f(k,j+1)$, and so both values will be generated for each vertex.

As noted above, the arrangement makes use of an indicator function. For triangular meshes, the indicator function i(q) for a point as vertex v'_q is given by, in the one-dimensional case

$$i\left(q\right) = L\left(q\right) - \left(L\left(q-1\right) + L\left(q+1\right)\right) + \frac{1}{2}\left(L\left(q-2\right) + L\left(q+2\right)\right) \tag{17}$$

and, for the two-dimensional case

$$i\left(q\right) = -6\,L\left(q\right) + 2\sum_{l \in N(1,0)\left(q\right)} L(l) - 2\sum_{l \in N(2,0)\left(q\right)} L(l) + \sum_{l \in N(2,1)\left(q\right)} L(l) \tag{18},$$

where the indices for the sums in equation (18) are in the neighborhood of the vertex for which the indicator function is being evaluated are as depicted in FIG. 4.

In generating the values of the indicator function for quadrilateral meshes, the arrangement makes use of a linear combination $L_{ind}(k,j+1)$ of the Laplacians $L_e(k,j+1)$ and $L_f(k,j+1)$ described above (equations (15) and (16), respectively), namely

$$L_{ind}\left(k, j+1\right) = 2\frac{K-5}{K-3}L_e\left(k, j+1\right) + \frac{1}{K-3}L_f\left(k, j+1\right) \tag{19}$$

for vertices for which valence "K" is not equal to "three." For vertices for which the valence "K" does equal "three," $L_{ind}(k,j+1)$ takes the form

$$L_{ind}\left(k, j + 1\right) = -\frac{22}{3} L_e\left(k, j + 1\right) + 2 L_f\left(k, j + 1\right) \tag{20}.$$

For regular vertices, that is, for vertices for which the valence "K" equals "four," equation (19) becomes

$$L_{ind}^{reg} = -2 L_e\left(k, j + 1\right) + L_f\left(k, j + 1\right) \tag{21}.$$

With this background, the indicator function i(k) for quadrilateral meshes is given by (omitting the index "j+1," since the indicator function will always be evaluated for vertices in the fine (j+1) level mesh)

$$i(k) = \frac{K}{4} L_{ind}(k) - \sum_{l \in N(1,0)(k)} L_{ind}(l) + \frac{1}{2} \sum_{l \in N(2,0)(k)} L_{ind}(l) + \sum_{l \in N(2,1)(k)} L_{ind}(l)$$
$$- \frac{1}{2} \sum_{l \in N(3,1)(k)} L_{ind}(l) - \frac{1}{2} \sum_{l \in N(3,2)(k)} L_{ind}(l) + \frac{1}{4} \sum_{l \in N(4,2)(k)} L_{ind}(l) \tag{22},$$

where the vertex neighborhoods for the respective sums are as shown in FIG. 5.

As noted above, the fine-to-coarse mesh generating arrangement generates a coarse ("j") level mesh from a finer ("j+1") level mesh. For a triangular mesh, the fine-to-coarse mesh generating arrangement determines, for each vertex v'_q located at position $c^{j+1}(k)$ in the finer ("j+1") level mesh for which a vertex v_q is to appear in the coarse ("j") level mesh, a position $c^j(k)$ for the vertex v_q in accordance with

$$c^J(k) = c^{j+1}(k) + \lambda L\left(k, j + 1\right) \tag{23},$$

where λ is a parameter whose value are determined as described below and L(k,j+1) is the value of the Laplacian generated for the vertex v'_q in the finer (j+1) level mesh. As noted above, not all of

1   the vertices in the finer ("j+1") level mesh will have corresponding vertices in the coarse ("j") level

2   mesh. If the value of the Laplacian L(k,j+1) generated for a vertex v'$_q$ in the finer ("j+1") level mesh

3   is relatively small, that is, if

4

$$\left| L\left(k, j+1\right) \right| < \varepsilon \tag{24}$$

5   for a selected small ε, then for equation (23), the position $c^j(k)$ of the corresponding vertex v$_q$ in the

6   coarse ("j") level mesh can be taken as the position $c^{j+1}(k)$ of the vertex v'$_q$. In that case, neither the

7   parameter λ, nor the indicator function i(q) (equations (17) and (18)), need to be generated.

8   If the value of the Laplacian L(k,j+1) is not small, that is, if the condition in equation (24)

9   does not hold, the manner in which parameter λ for use in equation (23) is to be determined will

10   depend on the value of the indicator function, generated as described above in connection with

11   equations (17) and (18) and whether the vertex is on a boundary, crease line or the like, a regular

12   vertex (that is, a vertex v'$_q$ whose valence "K" is "six") or an irregular vertex (that is, a vertex v'$_q$

13   whose valence "K" is other than "six"). If the value of the indicator function is zero, or close to zero,

14   for the vertex v'$_q$, a subdivision-inverse filter is used in which

      (i) if the vertex is on a boundary, crease line or the like

16

$$\lambda \; = \; -1 \tag{25},$$

17   (ii) if the vertex v'$_q$ is regular, that is, if its valence "K" is equal to "six," and is not on a

18   boundary, crease line or the like

19

$$\lambda \; = \; -\frac{3}{2} \tag{26}, \text{ and}$$

20   (iii) if the vertex v'$_q$ is irregular, that is, if its valence "K" is not equal to "six," and is not on

21   a boundary, crease line or the like

1

$$\lambda = \frac{8\,a(K)}{-5 + 8\,a(K)} \qquad (27),$$

2 where a(K) is as defined above in connection with equation (3).

3 On the other hand, if the value of the indicator function i(q) is not zero or close to zero, a

4 least-squares optimization is used in which

5 (i) if the vertex is on a boundary, crease line or the like

$$\lambda = \frac{1}{L(k)}\left[b_0^{1D}\, L(k) + \frac{1}{2} b_1^{1D}\left(L(k-1) + L(k+1) \right)\right] \qquad (28),$$

7 where $b_0^{1D} = -\dfrac{12}{35}$ and $b_1^{1D} = -\dfrac{23}{49}$.

8 (ii) if the vertex v'_q is regular, that is, if its valence "K" is equal to "six," and the vertex is not

9 on a boundary, crease line or the like

$$\lambda = \frac{1}{L(k)}\left[b_0^{reg}\, L(k) + \frac{1}{6} b_1^{reg} \sum_{l \in N(k,j+1)} L(l) \right] \qquad (29),$$

11 where $b_0^{reg} = -\dfrac{61}{5720}$ and $b_1^{reg} = -\dfrac{14403}{5720}$; it will be appreciated that the factor "$\dfrac{1}{6}$" in equation

12 (29) reflects the value of the valence "K," namely, "six."

13 (iii) if the vertex v'_q is irregular, that is, if its valence "K" is not equal to "six," and the vertex

14 is not on the boundary, crease line or the like

$$\lambda = \frac{1}{L(k)}\left[b_0^{irreg} \, L(k) + \frac{1}{K} b_1^{irreg} \sum_{l \in N(k,j+1)} L(l) \right] \tag{30},$$

1

2   where

3

$$b_0^{irreg} = \frac{2\left(5 - 8a(K)\right)\left(14647\,K - 391848\,a(K) + 391848\,a(k)^2\right)}{715\left(3 + 8\,a(k)\right)\left(256 + 41K - 512\,a(k) + 256\,a(k)^2\right)} \tag{31}$$

4   and

5

$$b_1^{irreg} = \frac{16\left(-5531K - 24521\,a(K) + 24521\,a(K)^2\right)}{715\left(256 + 41K - 512\,a(K) + 25\,a(K)^2\right)} \tag{32},$$

6   where, for both equations (31) and (32), "K" corresponds to the valence of the irregular vertex, and

7   a(K) is as defined in equation (3).

8   For a quadrilateral mesh, for each vertex v'_q located at position $c^{j+1}(k)$ in the finer (j+1) level

9   mesh, a vertex v_q is provided in the coarser (j) level mesh at a position $c^j(k)$ that is determined by

10

$$c^J(k) = c^{J+1}(k) + \lambda_1 \, L_e(k, j+1) + \lambda_2 \, L_f(k, j+1) \tag{33},$$

11   where λ_1 and λ_2 are parameters whose values are determined as described below, and $L_e(k,j+1)$ and

12    $L_f(k,j+1)$ are determined as described above in connection with equations (14) through (16). In the

13   one-dimensional case, that is, for vertices on a boundary, crease line or the like, since only Laplacian

14   L_e is used, only one parameter λ_1 need be generated for use in equation (33). On the other hand for

15   vertices that are not on a boundary, crease line, or the like, since both Laplacians L_e and L_f are used,

16   both parameters λ_1 and λ_2 are generated for use in equation (33). As with the triangular mesh, if the

1   values of $L_e(k,j+1)$ and $L_f(k,j+1)$ are relatively small, that is, if for a vertex v'_q in the fine ("j+1")

2   level mesh

3

$$\max\left(\left|L_e\left(k,j+1\right)\right|,\left|L_f\left(k,j+1\right)\right|\right) < \varepsilon \qquad (34)$$

4   for a selected small ε, then for equation (34), the position $c^j(k)$ of vertex v_q in the coarse ("j") level

5   mesh can be taken as the position $c^{j+1}(k)$ of vertex v'_q. In that case, neither the parameters λ_1 and λ_2,

6   nor the indicator function $i(k,j+1)$ (equation (22)), need to be generated.

7   If the condition in equation (34) does not hold, the values of parameter λ_1 and λ_2 for use in

8   equation (33) will depend on the value of the indicator function. For a vertex v'_q that falls on a

9   boundary, crease line or the like, the value of the parameter λ_1 will be generated in the same manner

10   as in a triangular mesh (reference equations (25) and (28) above). On the other hand, for a vertex

11   v'_q that does not fall on a boundary, crease line or the like, if the value of the indicator function

12    $i(k,j+1)$ generated therefor is near zero, that is, if

13

$$\left|i\left(k,j+1\right)\right| \leq \varepsilon \cdot \max\left(\left|L_e\left(k,j+1\right)\right|,\left|L_f\left(k,j+1\right)\right|\right) \qquad (35)$$

14   a subdivision-inverse filter is used in which, if the valence "K" of the vertex not equal to "three,"

15

$$\lambda_1 = -\frac{4}{K-3}$$
$$\lambda_2 = \frac{1}{K-3} \qquad (36),$$

16   and, if the valence "K" for the vertex is equal to "three,"

17

$$\lambda_1 = -8, \quad \lambda_2 = 2 \qquad (37).$$

1   On the other hand, if, for a vertex v'_q, the value of the indicator function $i(k,j+1)$ is not near zero, that

2   is, if the condition in inequality (35) is not satisfied,

3
$$\lambda_1 = \frac{1}{L_e(k,j+1)} \left[b_{10}^{cc} L_c(k,j+1) + \frac{1}{K} b_{11}^{cc} \sum_{l \in N(k,j+1)} L_e(l,j+1) \right] \qquad (38),$$

4   and

$$\lambda_2 = \frac{1}{L_f(k,j+1)} \left[b_{20}^{cc} L_f(k,j+1) + \frac{1}{K} b_{21}^{cc} \sum_{l \in N_f(k,j+1)} L_f(l,j+1) \right] \qquad (39),$$

6   which reflect a least squares optimization. If vertex v'_q is regular, that is, if the vertex's valence "K"

7   is "four,"

$$b_{10}^{cc} = -\frac{9946871}{4862025}$$

$$b_{11}^{cc} = -\frac{1024}{405}$$

$$b_{20}^{cc} = \frac{1644032}{972405} \qquad (40).$$

$$b_{21}^{cc} = -\frac{1338874}{972405}$$

9   On the other hand, if the vertex v'_q is irregular, that is, if the vertex's valence "K" is other than "four,"

$$b_{10}^{cc} = \frac{162307143936 - 92746939392\,K - 8924282387\,K^3}{4862025\left(12544 - 14336\,K + 4096\,K^2 + 901\,K^3\right)}$$

$$b_{11}^{cc} = \frac{1024\left(2793728 - 1596416\,K - 244001\,K^3\right)}{99225\left(12544 - 14336\,K + 4096\,K^2 + 901\,K^3\right)}$$

$$b_{20}^{cc} = \frac{512\left(-113305472 + 64745984K + 17391149\,K^3\right)}{4862025\left(12544 - 14336K + 4096\,K^2 + 901\,K^3\right)}$$

(41).

$$b_{21}^{cc} = \frac{4\left(8660934688 - 4949105536\,K - 1876158821\,K^3\right)}{4862025\left(12544 - 14336\,K + 4096\,K^2 + 901\,K^3\right)}$$

The following are approximations for equations (40) that are used in one embodiment of the arrangement:

$$b_{10}^{cc} = -2.04583$$

$$b_{11}^{cc} = -2.5284$$

(42),

$$b_{20}^{cc} = 1.69069$$

$$b_{21}^{cc} = -1.37687$$

5   and the following are approximations for equations (41) that are used in the same embodiment:

$$b_{10}^{cc} = \frac{1623.07 - 927.469\,K - 89.2428\,K^3}{609.892 - 697.02\,K + 199.149\,K^2 + 43.8068\,K^3}$$

$$b_{11}^{cc} = \frac{28.6078 - 16.3473\,K - 2.49857\,K^3}{12.4468 - 14.2249\,K + 4.06426\,K^2 + 0.894017\,K^3}$$

(43).

$$b_{20}^{cc} = \frac{-580.124 + 331.499\,K + 89.0427\,K^3}{609.892 - 697.02\,K + 199.149\,K^2 + 43.8068\,K^3}$$

$$b_{21}^{cc} = \frac{346.437 - 197.964\,K - 75.0464\,K^3}{609.892 - 697.02\,K + 199.149\,K^2 + 43.8068\,K^3}$$

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With this background, the operations performed by the fine-to-coarse mesh generating arrangement in generating a coarse ("j") level mesh from a finer ("j+1") level mesh will be described in connection with the flow charts in FIGS. 6 and 7. The flow chart in FIG. 6 depicts the operations performed in connection with a triangular mesh and the flow chart in FIG. 7 depicts the operations performed in connection with a quadrangular mesh. With reference initially to FIG. 6, the arrangement will first perform a series of steps to generate the Laplacian (equations (10) through (12)) for each of the points in the triangular finer ("j+1") level mesh. Initially, the arrangement will select a point in the finer (j+1) level mesh as vertex (step 100) and determine whether the point is regular, that is, whether its valence "K" is six, and does not fall on a boundary, crease line or the like (step 101). If the arrangement makes a positive determination in step 101, it proceeds to step 102

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in which it generates the Laplacian for the point selected in step 100 in accordance with equation (11).

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On the other hand, if the arrangement makes a negative determination in step 101, it will determine whether the point is irregular, that is, whether its valence "K" is less than six and does not fall on a boundary, crease line or the like (step 103). If the arrangement makes a positive determination in step 103, it proceeds to step 104 in which it generates the Laplacian for the point selected in step 100 in accordance with equation (12).

1   Finally, if the arrangement makes a negative determination in step 103, which will be the

2   case if the point falls on a boundary, crease line or the like, it proceeds to step 105 in which it

3   generates the Laplacian for the point selected in step 100 in accordance with equation (10).

4   Following step 102, 104 or 105, the arrangement will determine whether all of the points

5   have been selected in step 100 (step 106). If the arrangement makes a negative determination in step

6   106, it will not have generated Laplacians for all of the points in the finer ("j+1") level mesh, and,

7   in that case, it will return to step 100 to select another point and perform respective steps 101

8   through 105 in connection therewith. The arrangement performs step 100 and respective steps 101

9   through 105 through a plurality of iterations, in each iteration selecting and generating the Laplacian

10   for each of the points in the finer ("j+1") level mesh.

11   After all of the points have been selected and Laplacians generated therefor, the arrangement

12   will make a positive determination in step 106, at which point it will sequence to a series of steps

13   in which the arrangement determines the positions $c^j(q)$ for points in the finer ("j+1") level mesh that

14   are to be retained in the coarse ("j") level mesh. In those operations, the arrangement will select a

15   point in the finer ("j+1") level mesh (step 110) and determine whether the point is to be retained in

16   the coarse ("j") level mesh (step 111). If the arrangement makes a negative determination in step

17   111, the point is not to be retained in the coarse ("j") level mesh. In that case, the arrangement will

18   sequence to a step 112 in which it determines whether it has selected all of the points in the finer

19   ("j+1") level mesh. If the arrangement makes a negative determination in step 112, it will return to

20   step 110 to select another point in the finer ("j+1") level mesh. On the other hand, if the arrangement

21   makes a positive determination in step 112, which will be the case if it has processed all of the points

22   in the finer ("j+1") level mesh, it will exit (step 113). The arrangement can make use of any of a

23   number of methodologies in determining whether a point in the finer ("j+1") level mesh will be

24   retained in the coarse ("j") level mesh. For example, given a suitable point indexing methodology,

25   as will be apparent to those skilled in the art, the arrangement can determine whether a point in the

26   finer ("j+1") level mesh will be retained in the coarse ("j") level mesh.

27   Returning to step 111, if the arrangement makes a positive determination in that step, which

28   will be the case if the selected point in the finer ("j+1") level mesh will be retained, the arrangement

29   sequences to step 115, in which it determines whether the value of the Laplacian for the point

1 selected in step 110 is less than "ε" (reference inequality (24)). If the arrangement makes a positive

2 determination in step 115, it determines the position $c^j(q)$ of the point in the coarse ("j") level mesh

3 as the position $c^{j+1}(q)$ of the corresponding point in the finer ("j+1") level mesh (step 116).

4 Following step 116, the arrangement will sequence to step 112 to determine whether all of the points

5 in the finer ("j+1") level mesh have been processed, and, if not, return to step 110 to select another

6 point, and, if so, exit.

7 Returning to step 115, if the arrangement makes a negative determination in that step, it will

8 sequence to step 117, in which it will determine whether the point is regular, that is, whether its

9 valence "K" is six, and does not fall on a boundary, crease line or the like. If the arrangement makes

10 a positive determination in that step, it will generate a value for the indicator function (reference

11 equation (18)) (step 118). The arrangement will then determine whether the value of the indicator

12 function is zero or close to zero (step 119). If the arrangement makes a positive determination in

13 step 119, it will provide a value for the parameter λ as described above in connection with equation

14 (26) (step 120). On the other hand, if the arrangement makes a negative determination in step 119,

15 it will provide a value for the parameter λ as described above in connection with equation (29),

16 provided that the value of the parameter λ does not fall below the value specified in equation (26)

17 (step 121).

18 Returning to step 117, if the arrangement makes a negative determination in that step, it will

19 sequence to step 122 to determine whether the point is irregular, that is, whether its valence "K" is

20 not equal to six, and the point does not fall on a boundary, crease line or the like. If the arrangement

21 makes a positive determination in that step, it will generate a value for the indicator function

22 (reference equation (18)) (step 123). The arrangement will then determine whether the value of the

23 indicator function is zero or close to zero (step 124). If the arrangement makes a positive

24 determination in step 124, it will provide a value for the parameter λ as described above in

25 connection with equation (27) (step 125). On the other hand, if the arrangement makes a negative

26 determination in step 124, it will provide a value for the parameter λ as described above in

27 connection with equation (30), provided that the value of the parameter λ does not fall below the

28 value specified in equation (26) (step 126).

1   Returning to step 122, if the arrangement makes a negative determination in that step, it will

2   sequence to step 127 to determine whether the point falls on a boundary, crease line or the like. If

3   the arrangement makes a positive determination in that step, it will generate a value for the indicator

4   function (reference equation (17)) (step 128). The arrangement will then determine whether the

5   value of the indicator function is zero or close to zero (step 129). If the arrangement makes a

6   positive determination in step 129, it will provide a value for the parameter λ as described above in

7   connection with equation (25) (step 130). On the other hand, if the arrangement makes a negative

8   determination in step 129, it will provide a value for the parameter λ as described above in

9   connection with equation (28), provided that the value of the parameter λ does not fall below the

10   value specified in equation (25) (step 131).

11   Following step 120, 121, 125, 126, 130, or 131 , the arrangement will sequence to step 132,

12   in which it determines whether the value of the parameter λ generated in the respective step was

13   greater than "one," and, if so, limit the maximum value of parameter λ to "one" (step 133).

14   Thereafter, the arrangement uses equation (23) to determine the position $c^{j+1}(q)$ of the point in the

15   coarse ("j") level mesh (step 134).

16   Following step 134, the arrangement the arrangement will sequence to step 112 to determine

17   whether all of the points in the finer ("j+1") level mesh have been processed, and, if not, return to

18   step 110 to select another point, and, if so, exit.

19   The flow chart in FIG. 7 depicts the operations performed in connection with a quadrangular

20   mesh. With reference to FIG. 7, the arrangement will first perform a series of steps to generate the

21   Laplacians (equations (14) through (16)) for each of the points in the quadrilateral finer ("j+1") level

22   mesh. Initially, the arrangement will select a point in the finer (j+1) level mesh as vertex (step 150)

23   and determine whether the point does not fall on a boundary, crease line or the like (step 151). If

24   the arrangement makes a positive determination in step 151, it proceeds to step 152 in which it

25   generates the Laplacian for the point selected in step 150 in accordance with equations (15) and (16).

26   On the other hand, if the arrangement makes a negative determination in step 151, which will

27   be the case if the point falls on a boundary, crease line or the like, it proceeds to step 153 in which

28   it generates the Laplacian for the point selected in step 150 in accordance with equation (14).

1	Following step 152 or 153, the arrangement will determine whether all of the points have

2	been selected in step 150 (step 154). If the arrangement makes a negative determination in step 154,

3	it will not have generated Laplacians for all of the points in the finer ("j+1") level mesh, and, in that

4	case, it will return to step 150 to select another point and perform respective steps 151 through 153

5	in connection therewith. The arrangement performs step 150 and respective steps 151 through 153

6	through a plurality of iterations, in each iteration selecting and generating the Laplacian for each of

7	the points in the finer ("j+1") level mesh.

8	After all of the points have been selected and Laplacians generated therefor, the arrangement

9	will make a positive determination in step 154, at which point it will sequence to a series of steps

10	in which the arrangement determines the positions $c^j(q)$ for respective points in the coarse ("j") level

11	mesh using the positions $c^{j+1}(q)$ of the points in the finer ("j+1") level mesh and the values of

12	parameters to be generated and the Laplacians generated in steps 150 through 153. Initially, the

13	arrangement will select a point in the finer ("j+1") level mesh (step 160) and determine whether the

14	values of both Laplacians $L_e(k,j+1)$ and $L_f(k,j+1)$ generated for the point selected in step 160 are less

15	than "ε" (reference inequality (34)) (step 161). If the arrangement makes a positive determination

16	in step 161, it determines the position $c^j(q)$ of the point in the coarse ("j") level mesh as the position

17	$c^{j+1}(q)$ of the same point in the finer ("j+1") level mesh (step 162). Following step 162, the

18	arrangement will sequence to step 163 to determine whether all of the points in the finer ("j+1") level

19	mesh have been processed, and, if not, return to step 160 to select another point. On the other hand,

20	if the arrangement makes a positive determination in step 163, it will exit (step 164).

21	Returning to step 161, if the arrangement makes a negative determination in that step, it will

22	sequence to step 165, in which it determines whether the determine whether the point is regular, that

23	is, whether its valence "K" is four, and does not fall on a boundary, crease line or the like. If the

24	arrangement makes a positive determination in step 165, it will generate a value for the indicator

25	function (reference equation (22)) (step 166) and determine whether the value of the indicator

26	function is near zero (reference inequality (35)) (step 167). If the arrangement makes a positive

27	determination in step 167, it will generate values for λ_1 and λ_2 as described above in connection with

28	equations (36) (step 168). On the other hand, if the arrangement makes a negative determination in

29	step 167, it will generate values for λ_1 and λ_2 as described above in connection with equations (38)-

1   (40), in the process ensuring that the values of λ_1 and λ_2 are within predetermined limits $(-4 \le \lambda_1 \le 1/2,$

2    $1/4 \le \lambda_2 \le 1)$ (step 169).

3   Returning to step 165, if the arrangement makes a negative determination in that step, it will

4   sequence to step 170, in which it determines whether the point is irregular, that is, whether its

5   valence "K" is other than four, and does not fall on a boundary, crease line or the like. If the

6   arrangement makes a positive determination in step 170, it will generate a value for the indicator

7   function (reference equation (22)) (step 171) and determine whether the value of the indicator

8   function is near zero (reference inequality (35)) (step 172). If the arrangement makes a positive

9   determination in step 172, it determine whether the valence "K" of the point is "three" (step 173).

10   If the arrangement makes a positive determination in step 173, it will generate values for λ_1 and λ_2

11   as described above in connection with equation (37) (step 174). On the other hand, if the

12   arrangement makes a negative determination in step 173, it will generate values for λ_1 and λ_2 as

13   described above in connection with equation (36) (step 175). Returning to step 172, if the

14   arrangement makes a negative determination in step 172, which will be the case if the value for the

15   indicator function as generated in step 171 is not near zero, it will generate values for λ_1 and λ_2 as

16   described above in connection with equations (38)-(40), in the process ensuring that the values of

17   λ_1 and λ_2 are within predetermined limits $\left(-\dfrac{4}{K-3} \le \lambda_1 \le \dfrac{1}{2}, \dfrac{1}{4} \le \lambda_2 \le \dfrac{1}{K-3} \right)$ (step 176).

18   Returning to step 170, if the arrangement makes a negative determination in that step, it will

19   sequence to step 177 to determine whether the point falls on a boundary, crease line or the like. If

20   the arrangement makes a positive determination in that step, it will generate a value for the indicator

21   function (reference equation (17)) (step 178). The arrangement will then determine whether the

22   value of the indicator function is zero or near zero (reference inequality (35) above) (step 179). If

23   the arrangement makes a positive determination in step 179, it will provide a value for the parameter

24   λ as described above in connection with equation (25) (step 180). On the other hand, if the

25   arrangement makes a negative determination in step 179, it will provide a value for the parameter

26   λ as described above in connection with equation (28) (step 181).

1   Following step 168, 169, 174, 175, 180, or 181, the arrangement will sequence to step 182,

2   in which it uses equation (33) and values for parameters λ_1 and λ_2 generated in the previous steps to

3   determine the position $c^{j+1}(q)$ of the point in the coarse ("j") level mesh. Following step 182, the

4   arrangement will sequence to step 163 to determine whether all of the points in the finer ("j+1") level

5   mesh have been processed, and, if not, return to step 160 to select another point, and, of so, sequence

6   to step 164 to exit.

7   The arrangement provides a number of advantages. In particular, the invention provides an

8   arrangement for generating a coarse ("j") level mesh representing a surface, from a finer ("j+1") level

9   mesh surface representation. The arrangement makes use of an indicator function to provide an

10   indication as to which of several methodologies is be used at respective ones of the points in the finer

11   ("j+1") level mesh, including a subdivision-inverse filter methodology and a least-squares

12   optimization methodology.

13   It will be appreciated that numerous changes and modifications may be made to the

14   arrangement as described herein. For example, it will be appreciated that, although the arrangement

15   has been described in connection with meshes of triangular and quadrangular faces, it will be

16   appreciated that the arrangement may find utility in connection with meshes having faces of different

17   polygonal structures. In addition, although equations for generating the indicator functions and

18   parameters may differ than the specific ones described herein.

19   It will be appreciated that a system in accordance with the invention can be constructed in

20   whole or in part from special purpose hardware or a general purpose computer system, or any

21   combination thereof, any portion of which may be controlled by a suitable program. Any program

22   may in whole or in part comprise part of or be stored on the system in a conventional manner, or it

23   may in whole or in part be provided in to the system over a network or other mechanism for

24   transferring information in a conventional manner. In addition, it will be appreciated that the system

25   may be operated and/or otherwise controlled by means of information provided by an operator using

26   operator input elements (not shown) which may be connected directly to the system or which may

27   transfer the information to the system over a network or other mechanism for transferring

28   information in a conventional manner.

1   The foregoing description has been limited to a specific embodiment of this invention. It will

2   be apparent, however, that various variations and modifications may be made to the invention, with

3   the attainment of some or all of the advantages of the invention. It is the object of the appended

4   claims to cover these and such other variations and modifications as come within the true spirit and

5   scope of the invention.

6   What is claimed as new and desired to be secured by Letters Patent of the United States is: